Museum of Idaho
Rocky Mountain Adventure VI - Energy
The Wind Power Formula

Adapted from information found on the website of the American Wind Energy Association: http://www.awea.org/faq/windpower.html

Calculating the Amount of Power Available at a Given Wind Speed

Because air has mass and it moves to form wind, it has ***kinetic energy***. You may remember from science class that:
kinetic energy (joules) = 0.5 x m x V^2

where:
m = mass (kg) (1 kg = 2.2 pounds)
V = velocity (meters/second) (meter = 3.281 feet = 39.37 inches)
Usually, we're more interested in power (which changes moment to moment) than energy. Since **energy = power x time and density** is a more convenient way to express the mass of flowing air, the kinetic energy equation can be converted into a flow equation:

Power in the area swept by the wind turbine rotor:
P = 0.5 x rho x A x V^3

where:
P = power in watts (746 watts = 1 hp) (1,000 watts = 1 kilowatt)
rho = air density (about 1.225 kg/m^3 at sea level, less higher up)
A = rotor swept area, exposed to the wind (m^2)
V = wind speed in meters/sec (20 mph = 9 m/s) (mph/2.24 = m/s)
This yields the power in a free flowing stream of wind. Of course, it is impossible to extract all the power from the wind because some flow must be maintained through the rotor (otherwise a brick wall would be a 100% efficient wind power extractor). So, we need to include some additional terms to get a practical equation for a wind turbine.

Wind Turbine Power:
P = 0.5 x rho x A x Cp x V^3 x Ng x Nb

where:
P = power in watts (746 watts = 1 hp) (1,000 watts = 1 kilowatt)
rho = air density (about 1.225 kg/m^3 at sea level, less higher up)
A = rotor swept area, exposed to the wind (m^2)
Cp = Coefficient of performance (.59 {Betz limit} is the maximum thoretically possible, .35 for a good design)
V = wind speed in meters/sec (20 mph = 9 m/s)
Ng = generator efficiency (50% for car alternator, 80% or possibly more for a permanent magnet generator or grid-connected induction generator)
Nb = gearbox/bearings efficiency (depends, could be as high as 95% if good)

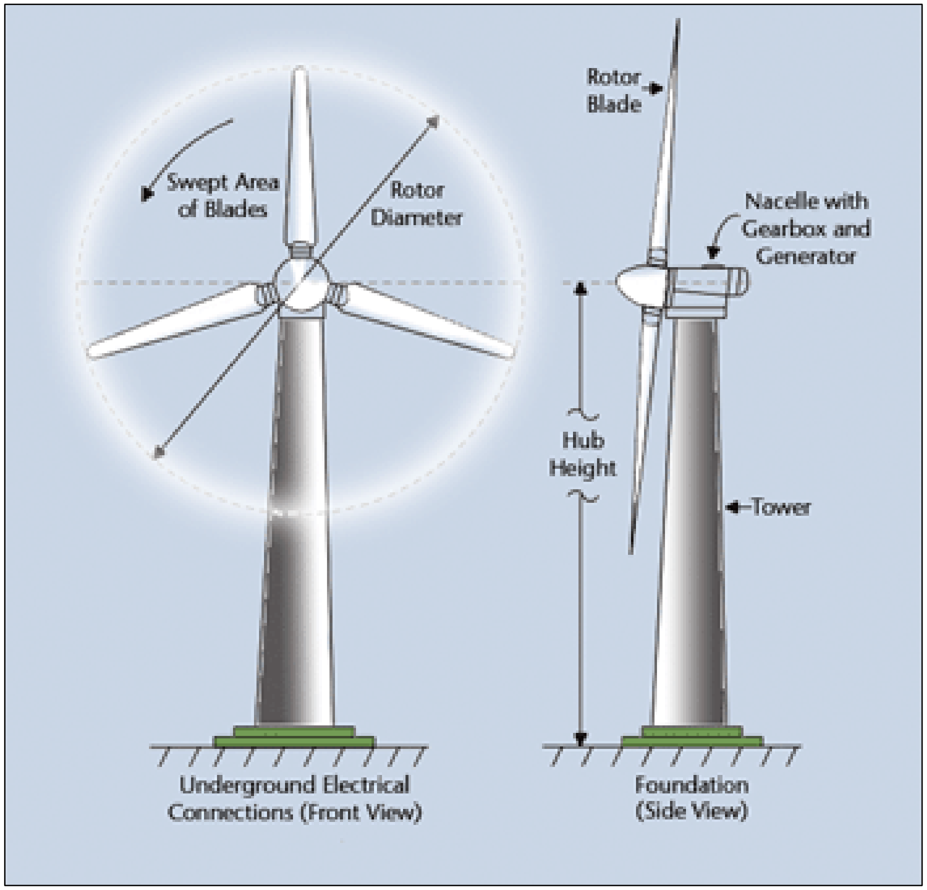
Fill in the empty blanks on the following chart: Use the wind power formula P = 0.5 x rho x A x Cp x V^3 x Ng x Nb

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Power (in watts)	**Rho** 50°F Air Density (kg/m^2)	**A** rotor swept area (m^2) (Area = Πr^2)	**Cp** Coefficient of Performance	**V^3** Wind speed in m/s (Mph/2.24=m/s)	**Ng** generator efficiency	**Nb** gearbox or bearing efficiency	**Energy** (in kilowatt hours) (Column 1/1000)	**Energy per Day** (Column 8 X 24)
	1.225 (sea level)	78	.35	5.4	.80	.80		
	1.010 (5000 ft)	78	.35	5.4	.80	.80		
	.796 (10000 ft.)	78	.35	5.4	.80	.80		
	1.010 (5000 ft)	78	.35	5.4	.70	.80		
	1.010 (5000 ft)	78	.35	5.4	.60	.80		
	1.010 (5000 ft)	78	.35	5.4	.50	.80		
	1.010 (5000 ft)	78	.35	6	.80	.80		
	1.010 (5000 ft)	78	.35	8	.80	.80		
	1.010 (5000 ft)	78	.35	10	.80	.80		
60	1.010 (5000 ft)		.35	10	.80	.80		
120	1.010 (5000 ft)		.35	10	.80	.80		
180	1.010 (5000 ft)		.35	10	.80	.80		
	1.010 (5000 ft)	78	.35	5.4	.80	.80		
	1.010 (5000 ft)	314	.35	5.4	.80	.80		
	1.010 (5000 ft)	1256	.35	5.4	.80	.80		

Use the above chart to answer the questions below. Work the problems on a separate of paper and show all work.

1. How does altitude affect the power generated by a turbine? Demonstrate with a graph.
2. How does generator efficiency affect the power generated by a turbine? Demonstrate with a graph.
3. How does wind speed affect the power generated by a turbine? Demonstrate with a graph)
4. How much does the swept area need to increase in order to double and triple the power output?
5. How does an increase in swept area increase the power generated by a turbine? Demonstrate with a graph.
6. Calculate how many kilowatt hours are produced each day if the winds powering the above turbines is a constant. (Put ans. in column 9)
7. The wind power turbines on the hills east of Idaho Falls (5000 feet above sea level) have a blade radius of 36.5 meters. How much power can they generate at wind speeds of 15 miles per hour? 25 miles per hour? 50 miles per hour? (Remember to change these velocities to meters per second)
8. The turbines cut out when wind speeds reach 62 miles per hour. Why?
9. Peak output for these turbines is 1.5 megawatts (1,500,000 watts). If the average home uses 3750 watts of power, how many homes can these turbines provide power for?
10. There are 43 turbines n the hill. What is the total peak output? How many homes can they provide power for?
11. If each home contains 4 people, the turbines east of Idaho Falls can supply power to how many people?

12. The Yerkels live in a house that is 2400 square feet with three bedrooms, two bathrooms and plenty of storage room. They use about 890 kilowatt/hours of energy per month. They live at sea-level. The coefficient of performance for their turbine will be .40. Average wind velocity in their neighborhood is 13 miles per hour. They should achieve a generator efficiency of about .75 because they will use a grid-connected induction generator. They plan to purchase a top-of-the-line generator with a gear-box efficiency of .90. How long will the blades need to be on the turbine they purchase?



Turbine sketch from:
http://www.daviddarling.info/encyclopedia/S/AE_swept_area.html